

A Praxeological Comparison of Quadratic Function Tasks in Indonesian and Singaporean Mathematics Textbooks Supporting High-Performing Students

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Abstract

This study addresses the gap in comparative praxeological analyses of mathematics textbooks, particularly concerning how curricular structures influence opportunities for high-performing students to develop advanced mathematical thinking. The study examines the techniques, technologies, and task variations in Indonesian and Singaporean textbooks on quadratic functions and identifies potential learning obstacles shaped by curriculum design. Using a qualitative document analysis, data were collected from two widely used national textbook and analyzed through the Anthropological Theory of the Didactic (ATD), focusing on task types, techniques, epistemological ordering, and differentiation for high-performing students. The results reveal that Singapore's curriculum, supported by Subject-Based Banding, enables textbook to classify tasks into basic, intermediate, and advanced levels, thus providing systematic support for high-performing learners and fostering symbolic abstraction. In contrast, the Indonesian textbook presents mixed-level tasks without explicit differentiation, and introduces quadratic equations before quadratic functions, which may generate didactical and epistemological obstacles, especially for students capable of deeper abstraction. The findings imply that curricular decisions profoundly shape textbook design and determine the extent to which high-performing students can access cognitively demanding tasks. This study underscores the need for stronger alignment between assessment, curriculum, and textbook design in Indonesia to better support differentiated learning and promote relational understanding among all learners, including high-performing students.

Keywords: comparative study, high-performing mathematics, mathematics textbook, praxeology, quadratic function.

Introduction

Textbooks are one of the main learning resources in mathematics learning because they function as a source of material, a guide for thinking activities, and the main reference for teachers and students (Sugiarni et al., 2024). The form and quality of presentation in textbooks can influence the type of cognitive activities students engage in, whether students are invited to perform routine procedural tasks, contextual application tasks, or open-ended tasks that challenge high-level reasoning so the role of textbooks is critical in determining the mathematical learning opportunities available to all students (Anggraena, 2019). This is in line with various studies that show that the quality of assignments and activities in textbooks greatly affects the depth of understanding of concepts and the development of

students' mathematical abilities (Schubring & Fan, 2018; Wang & Fan, 2021). Based on this, the comparison of textbooks from different countries has become a field of study that continues to grow, especially because each country has a different curriculum philosophy, material presentation structure, and pedagogical orientation (Utami et al., 2025). These differences can be analyzed to see if they can be adopted and applied as a textbook evaluation of a country, especially the topic of quadratic functions.

The topic of quadratic functions itself is a fundamental topic in algebra because it is the basis for advanced concepts such as polynomial functions, optimization, calculus, and modeling of real phenomena (Kieran, 2022). A strong mastery of quadratic functions allows students to develop high-level mathematical representation, generalization, and reasoning skills. Therefore, the quality of the presentation of quadratic function tasks in textbooks is a crucial aspect for the development of students' mathematical competence at various levels of ability.

A number of comparative textbook studies, particularly those comparing Indonesian textbooks with those from other countries, have revealed differences in content organization, conceptual depth, and the characteristics of the tasks presented. However, most of these studies have primarily focused on general aspects of textbook comparison, such as levels of cognitive demand, task contexts, or representational variations, without examining how textbooks address the learning needs of students with high mathematical ability. In fact, high-ability students require specific instructional support, including tasks that involve higher cognitive complexity, opportunities for exploration, and activities that promote generalization and mathematical creativity (Elgrably & Leikin, 2021; Leikin & Elgrably, 2022). Previous research also indicates that when textbooks fail to provide sufficiently challenging tasks, high-achieving students may experience underachievement and diminished motivation to fully develop their mathematical potential. (Rotigel & Fello, 2004; Smedsrud et al., 2022).

In the Indonesian context, several studies have reported that mathematics textbooks tend to prioritize the fulfillment of minimum curricular requirements, resulting in a limited presence of tasks that demand high levels of reasoning. In contrast, textbooks from countries such as Singapore are widely recognized for placing greater emphasis on non-routine problem solving and the development of conceptual structures (Hendriyanto et al., 2023; Kuncoro et al., 2024; Purnomo et al., 2024). However, there has been no research that

specifically compares how the two textbooks facilitate highly capable students on a particular topic, specifically quadratic functions, using a praxeological framework.

The praxeological framework allows for a more in-depth analysis of the task structure (T), expected technique (τ), technology or the reason behind the technique (θ), and the theory behind it (Θ) (Chevallard, 2019). Thus, this approach can uncover how textbooks provide opportunities for students to undertake more complex, creative, and reflective mathematical activities. Based on this gap, the present study examines quadratic function tasks in Indonesian and Singapore mathematics textbooks to investigate the extent to which these textbooks facilitate the learning needs of students with high mathematical ability. This research is expected to contribute new insights into the quality of task design in mathematics textbooks and its role in supporting the optimal development of students' mathematical potential.

Method

This study uses a qualitative research design with a hermeneutic document-analysis approach. The design of this research allows researchers to trace experiences in the form of the results of human thinking activities such as the understanding of knowledge expressed in the form of writing such as textbooks or student worksheets. Meanwhile, hermeneutics will explain the interpretation of existing phenomena both verbal and non-verbal. The phenomenon to be studied is the tasks of the topic of quadratic functions in mathematics textbooks in Indonesia and Singapore that can facilitate students with high mathematical abilities.

The subjects of this study were 49 grade XI students in one of the private high schools in Bandung. The instrument used was a test designed to measure cognitive abilities according to the revised edition of the bloom taxonomy such as understanding (C2), applying (C3), and analyzing (C4) (Netriwati, 2018). The design of this test question is based on tracing students who have the potential to have high mathematical ability and is adjusted to the the topic of quadratic functions that has been studied and the textbooks used.

The Indonesian mathematics textbook analyzed in this study is the official mathematics textbook published by the Ministry of Research and Technology, entitled Matematika untuk SMA/SMK Kelas X (Edisi Revisi) tahun 2023 (Susanto et al., 2023). This book was chosen because it can be accessed for free by all teachers and students so that it

can be a representation of textbooks that are widely used by teachers and students in Indonesia. Meanwhile, Singapore's mathematics textbook is a textbook titled Think! Additional Mathematics (New Syllabus Additional Mathematics 10 th Edition (2020-2024)) published by one of the private publishers in Singapore namely Shing Lee Publishers PTE LTD (Heng et al., 2020). This book was chosen because this book is one of the most widely used textbooks in Singapore schools, besides that this book is intended for high school students equivalent to grade 9 or grade 10 in Indonesia who have high mathematical ability, namely G2/G3.

Table 1. Comparison of the Number of Tasks
on the Topic of Square Functions in Indonesian and Singapore Textbooks

Sub-Topic	IB	SB	Sub-Topic
5.1 Characteristics quadratic functions	10	I (8)	1.1 Quadratic function of the form $y = a(x - p)(x - q)$
5.2 Constructing the quadratic function	3	IV (23)	1.2 Quadratic function of the form $y = a(x - h)^2 + k$
5.3 Solving the problem of quadratic functions	3	I (2)	1.3 Condition for quadratic curve to lie completely above or below X-axis
Competency Test	10	I (2)	1.4 Quadratic function in real world contents
		7	Exercise
		3	Review Exercise
			Challenge Yourself

The two textbooks will be analyzed based on a praxeological framework that has four components, namely tasks, techniques, technology and theory. The results of this analysis will be associated with the criteria of the students' mathematical ability so that it will be seen whether the tasks presented in the textbook have facilitated the students with special needs. Based on Danek (2018), there are two criteria in determining tasks that can facilitate students with high mathematical ability, namely 1) tasks that make students think about determining

appropriate techniques; 2) solutions require one or more technologies or several theories (Danek, 2018). This task is often called the ill-structured problem type. Ill-structured problems usually have unclear problem statements, have diverse solutions and contain uncertainty about which concepts, rules, and principles are needed to solve the problem. This provides four different stages of insight for students who have high mathematical ability, namely 1) spontaneous insight; 2) passive gradual insight; 3) sudden insight; 4) active gradual insight (Stenberg et al., 2024). However, because the characteristics of students in several high schools in Indonesia are diverse, students' high mathematical ability is based on the cognitive level of the revised edition of the bloom taxonomy starting from the ability to analyze (C4).

Furthermore, data triangulation will be carried out in the form of ill-structured quadratic function tasks and will be further explained based on literature review.

Results and Discussion

Based on the results of the research that has been carried out, there are two findings, including the results of students' mathematical comprehension ability and the results of the praxeological analysis of Indonesian and Singapore mathematics textbooks as follows:

Table 2. Students' Mathematical Cognitive Level

Cognitive Level	Number of Students
C2	37
C3	8
C4	4

Table 2 shows that most of the students' levels of mathematical understanding of the the topic of quadratic functions only reach the level of cognitive understanding (C2). This result often makes teachers focus only on students' low mathematical abilities, while the needs of students with high mathematical abilities often go unnoticed by teachers. In fact, they also have the right to develop and not always wait for other students.

Table 3. Analisis Praxis-Block of Singaporean Textbook

Type of Task (T_{SB})	Task (t_{SB})	Technique (τ_{SB})
T_{SB1} : Understanding how state the coordinates of the turning point of quadratic function	t_{SB1} : Find the maximum or minimum value from each of the following functions with or without sketching the graph. a) b)	τ_{SB1} : visuospatial (graphing) $\tau_{SB4,1}$: symbolic (factorizing)

Type of Task (T_{SB})	Task (t_{SB})	Technique (τ_{SB})
	c)	$\tau_{SB4,2}$: symbolic
	d)	(completing the square)
	t_{SB2} : Express each of the following expression in the form $a(x - h)^2 + k$	τ_{SB2} : formal
	a)	
	b)	
	c)	τ_{SB2} : formal
	d)	
	e)	
	f)	
	t_{SB3} : a) Explain why $5(x + 1)^2 - 6$ can never be less than -6.	
	b) the maximum value of $\frac{1}{2} - 4(x - 7)^2$ is $\frac{1}{2}$. Do you agree? Explain why?	τ_{SB1} : visuospatial (graphing)
	c) find the value of x for which the maximum of $\frac{1}{2} - 4(x - 7)^2$ occurs.	$\tau_{SB4,2}$: symbolic (completing the square)
	t_{SB4} : Find the maximum or minimum value from each of the following functions with or without sketching the graph.	
	a)	
	b)	
	c)	τ_{SB3} : numerical
	d)	$\tau_{SB4,1}$: symbolic (factorizing)
	e) $y = (6x + 1)(x - 5) - 1$	$\tau_{SB4,2}$: symbolic (completing the square)
	f) $y = -3(x + 4)^2 - 5$	
	t_{SB5} : The maximum value of the expression $ax^2 + 10x + c$ is 7. Find a pair of possible values of a and c.	
T_{SB2} : Understanding what condition for quadratic curve to lie completely above or below x-axis	t_{SB6} : Does each of the following curves lie completely above or below the x-axis? Explain your answer.	τ_{SB1} : visuospatial (graphing)
	a) $y = -x^2 + 10x - 31$	τ_{SB2} : formal
	b) $y = \frac{1}{4}x^2 + 2x + 7$	$\tau_{SB4,2}$: symbolic (completing the square)
T_{SB3} : Understanding a situation using quadratic function	t_{SB7} : Jump of kangaroo. The jump of kangaroo can be modelled by the equation $y = -0,125(x - 4)^2 + 2$, where x metres is the horizontal distance and y metres is the corresponding height.state the greatest height that the kangaroo can jump and the corresponding horizontal distance.	τ_{SB1} : visuospatial (graphing)
	t_{SB8} : Profit function. The profit, \$y, of accompany can be approximated by $y = -\frac{1}{20}x^2 + 75x - 600$, where x is the number of goods sold.	$\tau_{SB4,2}$: symbolic (completing the square)
	(i)Explain the meaning of “-600” in the equation.	
	(ii)How many goods must be sold in order for the company to yield the maximum profit? What is this maximum profit?	τ_{SB1} : visuospatial (graphing)
	t_{SB9} : Shape of a skateboard ramp. The diagram shows part of the cross section of a skateboard ramp. The height, y metres, of the ramp can be modelled by the equation $y = \frac{9}{80}x^2 - \frac{13}{5}x$, where x is the horizontal distance from the point O in metres.	$\tau_{SB4,2}$: symbolic (completing the square)

Type of Task (T_{SB})	Task (t_{SB})	Technique (τ_{SB})
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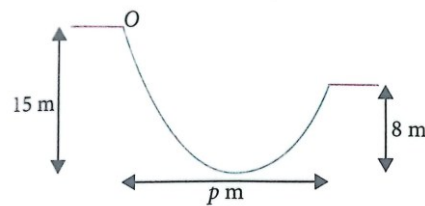


Figure 1. Illustration of t_{SB9}

(i) Express $\frac{9}{80}x^2 - \frac{13}{5}x$ in the form $a(x - h)^2 + k$.

(ii) A skateboarder has the greatest amount of kinetic energy at the lowest point of the ramp. Find the horizontal distance he has travelled from the starting point O to the lowest point of the ramp.

(iii) Find p, where p m is the width of the ramp.

τ_{SB1} : visuospatial (graphing)
 $\tau_{SB4,2}$: symbolic (completing the square)

t_{SB10} : Petrol consumption. A group of students wanted to find out the relationship between the amount of petrol consumed by a car and its speed. The table shows the data they obtained.

Speed (x km/h)	30	42	55	62	70	76	81	90
Petrol (y litres / 100 km)	7.6	6.8	6.2	6.1	6.0	6.0	6.1	6.4

Figure 2. Table of the task t_{SB10}

a) Is the equation $y = 0,001x^2 - 0,14x + 10,9$ a good model for the data? Explain your answer.

b) Based on the data given in the table, Vani thinks that the car consumes the least amount of petrol when it travels at 73 km/h.

(i) By expressing $y = 0,001x^2 - 0,14x + 10,9$ in the form $a(x - h)^2 + k$, explain whether you agree with her.

(ii) Hence, write down the optimal speed the car should travel at to minimize the amount of petrol consumed.

The types of assignments in the Singapore textbook have been categorized based on the student's cognitive level, namely basic, intermediate, and advanced levels. In addition, there is an 'open' code in one of the assignments which is an open-ended question type so that it allows many alternative answers. In addition, the numbers used are quite diverse, ranging from whole numbers, fractions, to decimal numbers. This shows that the application of quadratic function matter is more approximate so that the mathematical model does not always have an integer coefficient.

Table 4. Analisis Praxis-Block of Indonesian Textbook

Type of Task (T_{IB})	Task (t_{IB})	Technique (τ_{IB})
T_{IB1} : Understanding the	t_{IB1} : Determine the parabola that opens upward and downward. Compare the two parabolas. Which parabola do you think has a	τ_{IB1} : visuospatial

Type of Task (T_{IB})	Task (t_{IB})	Technique (τ_{IB})
characteristic of quadratic function	wider opening? Which constant in the quadratic function $y = f(x) = ax^2 + bx + c$ determines the opening of a parabola?	



Figure 3. Bridge in task t_{IB1}

t_{IB2} : Based on the following function, determine the quadratic function that opens upward and downward is.

- $f(x) = 3x^2 + 4x + 1$
- $f(x) = -4x^2 + 4x + 5$

t_{IB3} : You notice that the starting position doesn't start at zero.

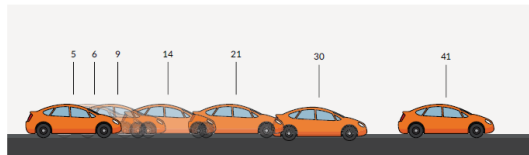


Figure 4. Cars in task t_{IB3}

- Fill in the car mileage table against time.

Waktu (det)	0	1	2	3	4	5	6
Jarak (m)							

Figure 5. Table in task t_{IB3}

- Draw a distance graph against time on a square piece of paper.
- Does the result depict the shape of a parabola?
- What is the value of c when referring to $y = f(x) = ax^2 + bx + c$?

t_{IB4} : The table below shows the mileage of a car at any given time.

Waktu (Detik)	0	1	2	3	4	5	6
Jarak (m)	0	5	8	9	8	5	0

Figure 6. Table in task t_{IB4}

Without drawing a graph, determine whether the square function graph is open up or down. Explain your reasoning.

t_{IB5} : Define multiple cutoff points by the x-axis of the following quadratic function.

- $f(x) = 3x^2 + 4x + 1$
- $f(x) = -4x^2 + 4x + 5$

t_{IB6} : Determine the coordinates of the turning point, the axis of symmetry, the coordinates of the cut-off point with the y-axis,

τ_{IB1} : visuospatial (graphing)
 τ_{IB2} : formal

τ_{IB1} : visuospatial
 τ_{IB3} : numerical

τ_{IB1} : visuospatial
 τ_{IB3} : numerical

τ_{IB1} : visuospatial
 τ_{IB2} : formal

τ_{IB1} : visuospatial
 τ_{IB2} : formal

Type of Task (T_{IB})	Task (t_{IB})	Technique (τ_{IB})
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and the many cut-off points from the graph of the square functions below.

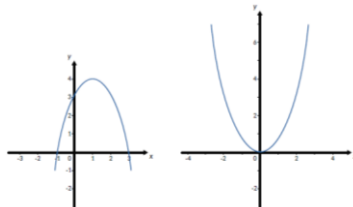


Figure 7. Graph of quadratic function

Is the relationship between the turning point and the graph open or down?

τ_{IB1} : visuospatial
 τ_{IB2} : formal
 τ_{IB3} : numerical

τ_{IB1} : visuospatial

t_{IB7} : Take a look at the table below, which shows the mileage of a car as a function of time.

Waktu (detik)	0	1	2	3	4	5	6
Jarak (m)	8	13	16	17	16	13	8

Figure 8. Table in task t_{IB7}

τ_{IB1} : visuospatial

- What is the maximum distance traveled?
- What is the maximum point coordinate?
- Determine the equation of the symmetrical axis line.

t_{IB8} : For each case below specify whether the square function graph opens up or down.

- Production costs as a function of the quantity of goods.
- The sound quality of the sound system as a function of the amplitude of sound waves.
- The effectiveness of the drug as a function of the drug dose.

t_{IB9} : Which of the following graphs is a graph of quadratic functions?

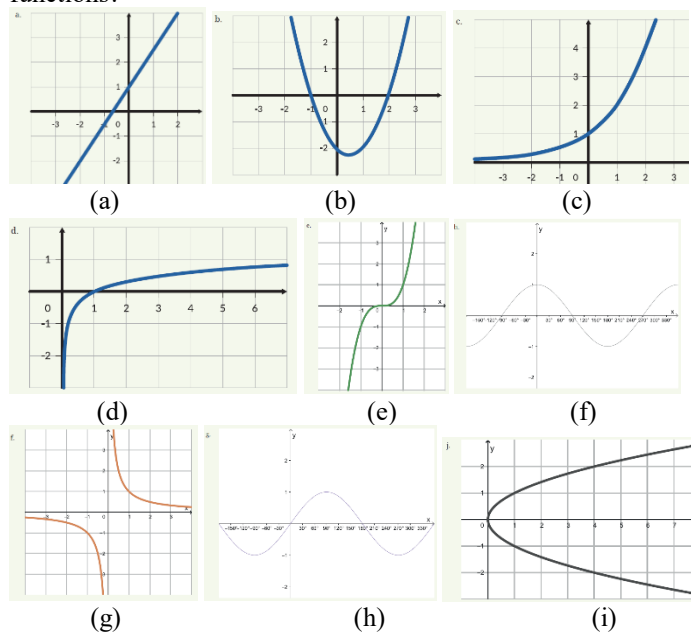


Figure 9. Some graphs in task t_{IB9}

Type of Task (T_{IB})	Task (t_{IB})	Technique (τ_{IB})																		
T_{IB2} : Constructing a quadratic function form	t_{IB9} : Square function with turning point (2,6) and through the point (1,7). Express the quadratic function in all three forms.	τ_{IB1} : visuospatial τ_{IB2} : formal																		
	t_{IB10} : A ball is thrown from an initial height of 4 m and reaches a maximum height of 8 m after two seconds from being thrown. Specify the quadratic function in all three forms.	$\tau_{IB4,(1,2,3)}$: symbolic																		
T_{IB3} : Solve the problem using quadratic function	t_{IB11} : The table below shows the relationship between car speed and fuel efficiency. Take a graphic approach of a quadratic function where $P(x)$ is the fuel economy and x is the speed of the car.	τ_{IB1} : visuospatial τ_{IB3} : numerical																		
	<table border="1"> <thead> <tr> <th>Speed (km/jam)</th> <th>16</th> <th>32</th> <th>48</th> <th>64</th> <th>80</th> <th>96</th> <th>112</th> <th>128</th> </tr> </thead> <tbody> <tr> <th>Fuel (km/liter)</th> <td>7,5</td> <td>10,2</td> <td>12,2</td> <td>13,2</td> <td>13,5</td> <td>12,8</td> <td>11,3</td> <td>9,1</td> </tr> </tbody> </table> <p>What is the speed that results in maximum savings?</p>	Speed (km/jam)	16	32	48	64	80	96	112	128	Fuel (km/liter)	7,5	10,2	12,2	13,2	13,5	12,8	11,3	9,1	τ_{IB1} : visuospatial $\tau_{IB4,(1,2,3)}$: symbolic
	Speed (km/jam)	16	32	48	64	80	96	112	128											
Fuel (km/liter)	7,5	10,2	12,2	13,2	13,5	12,8	11,3	9,1												
t_{IB12} : Scientists measure the concentration of drugs in the blood over time. The quadratic function is used to model this, i.e. $C(t) = -0,0003t^2 + 0,09t$ with the t -unit is minutes and the concentration of the drug in mg/L. Determine how long it takes for the drug concentration to be zero and the maximum concentration of the drug in the blood.	τ_{IB1} : visuospatial τ_{IB2} : formal																			
t_{IB13} : Notice the table below which shows the relationship between the selling price of a pen and the number of pens sold.	<table border="1"> <thead> <tr> <th>Harga Satu Pulpen (p)</th> <th>Jumlah Pulpen yang Terjual (J(p))</th> </tr> </thead> <tbody> <tr> <td>3000 rupiah</td> <td>2000</td> </tr> <tr> <td>2700 rupiah</td> <td>1800</td> </tr> <tr> <td>2400 rupiah</td> <td>1600</td> </tr> </tbody> </table>	Harga Satu Pulpen (p)	Jumlah Pulpen yang Terjual (J(p))	3000 rupiah	2000	2700 rupiah	1800	2400 rupiah	1600	τ_{IB3} : numerical $\tau_{IB4,(1,2,3)}$: symbolic										
Harga Satu Pulpen (p)	Jumlah Pulpen yang Terjual (J(p))																			
3000 rupiah	2000																			
2700 rupiah	1800																			
2400 rupiah	1600																			

Figure 10. Table in task t_{IB13}

The cost of making one pen is Rp. 2000.00. Specify:

- The function $J(p)$ which expresses the relationship between the price of a pen and the sale of a number of pens,
- The profit function which is the difference between the selling price and the cost of making a number of pens, and
- The selling price of a pen to get the biggest profit.

Indonesian textbooks generally do not categorize assignments according to students' cognitive levels. Therefore, teachers are required to possess the ability to identify and select task types that are appropriate for the cognitive levels of students in their classrooms. Moreover, the dominance of low-cognitive-level tasks in these textbooks positions them as the primary reference for instructional design, which in turn leads to classroom practices that tend to confine students to the completion of low-level tasks (Khawlhing et al., 2025).

Based on block praxis, it can be seen that numerical and formal techniques still dominate compared to symbolic techniques. This shows that Indonesian textbooks still focus

on shaping students' understanding through visuospatial, formal and numerical so that the process of abstraction, namely with symbolic techniques, is very limited.

Table 5. Analisis Logos-Block

Technology (θ_{IB})	Theory (Θ_{IB})	Technology (θ_{SB})	Theory (Θ_{SB})
θ_{IB1} : graph in cartesius coordinate, figure	Θ_{IB1} : Transformation	θ_{SB1} : graph in cartesius coordinate	Θ_{SB1} : Transformation
θ_{IB2} : quadratic function form	Θ_{IB2} : Quadratic Function	θ_{SB2} : quadratic inequality and quadratic function form and equivalence	Θ_{SB2} : Quadratic Function
IB3: substitution the number and perform arithmetic operations	Θ_{IB3} : Sistem Persamaan Linear Tiga Variabel	θ_{SB3} : substitution the number	
θ_{IB4} : factorizing, completing the square, and using the quadratic formula		θ_{SB4} : factorizing and completing the square	

Based on the analysis of logos blocks, it can be seen that the difference in technology used by Indonesian textbooks and Singapore textbooks is mainly in theory. Theory Θ_{IB3} is used to construct algebraic forms of quadratic functions and this theory is not used in Singapore books.

Table 6. Quadratic Function Tasks with High Category Cognitive Level

Cognitive Level	T_{IB}	T_{SB}
C4	t_{IB8}, t_{IB810}	$t_{SB3}, t_{SB7}, t_{SB8}$
C5	t_{IB12}	$t_{SB5}, t_{SB9}, t_{SB10}$
C6	t_{IB13}	

Based on table 5, it can be seen that assignments in Indonesian textbooks provide opportunities for students with high mathematical ability to complete assignments with cognitive levels C4, C5 and C6 while in Singapore textbooks only provide assignments with cognitive levels C4 and C5 in this chapter. This shows that both textbooks seek to provide assignments that can facilitate students with high mathematical abilities.

The results of the study showed two main findings, namely (1) the level of students' mathematical ability which tends to be at a low cognitive level, and (2) the characteristics of the different praxeological characteristics of Indonesian and Singapore textbooks in providing a variety of tasks, techniques, technologies and theories. These findings provide important implications for the learning of quadratic functions, especially in facilitating high-ability students (Stacey, 2011; Wijaya, 2016).

Low Levels of Mathematical Comprehension of Students and Implications for High-Ability Students

Most students are at the C2 comprehension level (comprehension), while only a small fraction reach C3-C4. This condition is in accordance with the previous national finding that most Indonesian students are still at the procedural level and rarely achieve a higher relational or conceptual understanding. In the context of high-ability students, C2 dominance has the potential to pose an underchallenge because students with high abilities require tasks that demand reasoning, generalization, and explanation of non-routine problems (Leikin, 2011). According to the theory of mathematical giftedness, highly gifted students thrive when faced with complex, open, and demanding representational flexibility (Sheffield, 2017). If this is not facilitated, students with high abilities will feel bored and even annoy other friends (Smedsrud et al., 2022). Students like this are worried that they will lose their motivation to learn so that they will decrease their mathematical skills.

$$\begin{aligned}
 & 2) \quad 2(p+1) = 12x \quad 1 = p \times l \\
 & \quad 2(p+2x) = 12x \quad = 4 \times 2x \\
 & \quad p+2x = 6x \\
 & \quad p = 6x - 2 \\
 & \quad \quad = 4x \\
 \\
 & b \quad 4s = 80 - 12x \\
 & \quad s = \frac{80 - 12x}{4} = 20 - 3x \\
 \\
 & c \quad 1p = 9^2 = (20 - 3x)^2 \\
 & \quad \quad = 400 - 120x + 9x^2
 \end{aligned}$$

Figure 11. Modeling of a problem by one of the students with high mathematical ability

Comparison of Praxeology Indonesian and Singaporean Textbook and Its Impact on High-Performing Students

Basically, the approach (praxeology) of the two textbooks is not much different, it's just that the presentation of the topic of quadratic functions in Singapore textbooks is adjusted to the respective national curriculum. Singapore's mathematics textbooks are superior in facilitating students' diverse abilities, as students are categorized first through their Subject-Based Banding system (subject comparison), so that students' mathematical abilities are divided into G1, G2/G3 Mathematics, and G2/G3 Additional Mathematics levels.

In fact, for students with high abilities or special interest in mathematics, namely those who take G2/G3 Additional Mathematics, the textbook also provides assignments with three more levels: basic, intermediate, and advanced. This system shows that educational

institutions in Singapore have taken an active role in providing information about the abilities of each student in each subject, so that potential learning obstacles can be anticipated from both ontogenic, didactical, and epistemological aspects (Lindorff et al., 2019).

In contrast to the curriculum in Indonesia, where assessment through national exams is now called national assessment is used as a tool to measure student understanding (numeracy) in general (Eti Tamsiyati et al., 2025). This makes the individual understanding of each student not clearly defined. As a result, the final assessment remains up to the teacher. Meanwhile, teachers are not necessarily able to describe students' abilities objectively through the scores of assessments prepared by their respective schools, because there is no binding national standard (Ping et al., 2026). Therefore, when students continue their education to the next stage, teachers in advanced schools must perform diagnostic tests again. The effect of this system of curriculum and assessment affects textbook writing in Indonesia, textbooks tend not to categorize the difficulty of assignments, as this is returned to the teacher. Teachers are expected to be able to adjust the content according to the cognitive abilities of each student, so that teachers in Indonesia individually are expected to be able to anticipate potential learning obstacles from textbooks, especially mathematics.

In addition, the effect of the curriculum is also seen in the presentation of mathematics content in textbooks, especially the topic on quadratic functions. In the Singapore curriculum, the topic on quadratic functions and quadratic equations is separated, the concept of quadratic functions is introduced in Secondary 2, while the material on quadratic equations is discussed later in Secondary 3. This presentation allows students to learn the function first, and then delve into the equations. On the other hand, in Indonesia, the topic on quadratic equations is taught first, then followed by the topic on quadratic functions, both are in the same chapter and are taught in class X.

In theory, a quadratic function is a function meaning that the output $f(x)$ can have many possibilities for different values of x , whereas quadratic equations produce a specific value of y or $f(x)$ based on x . Epistemologically, ideally, students understand the concept of function first before learning equations. If the learning order is reversed, this can lead to didactical obstacles. For example, in the study, it was stated that in functions such as $y = 3x^2 - 9x + 6$, students tend to divide the entire equation by 3 for the sake of simplifying so that it ends with $y = x^2 - 3x + 2$ as if to think of it as an ordinary quadratic equation (Huang et al.,

2012). It illustrates that formal knowledge (equations) is introduced before general understanding (function), which can interfere with students' conceptual understanding.

Based on the results of the research, it can be seen that Singaporean textbooks at this stage have led students to the process of abstraction such as the use of symbolic techniques dominates (Nurhikmayati et al., 2025). Meanwhile, in Indonesian textbooks, numerical and formal techniques still dominate. This condition indicates that Indonesian textbooks present more assignments for students with low and medium ability (Yang & Sianturi, 2017), while assignments for high-ability students are very few even though research shows that there are indeed few students who fall into the category of "high ability". As a result, mathematics teachers in Indonesia have to work extra hard, starting from anticipating potential learning obstacles that arise from textbooks to adjusting the way knowledge is delivered to students.

Conclusion

The results of the praxeology analysis of the two mathematics textbooks show that the main difference does not lie in the theoretical structure or type of assignment, but in the way the content is presented which is directly influenced by the curriculum policies of each country. The Singapore curriculum, which implements Subject-Based Banding (SBB), provides information on students' abilities in a more structured manner that allows textbooks to clearly present a variety of task levels ranging from basic, intermediate to advanced. This helps minimize learning obstacles such as epistemological, didactic, and ontogenic because students have learned based on relatively homogeneous levels of ability.

On the other hand, in Indonesia, national assessments are not used to classify individual abilities, so teachers must re-diagnose students' abilities at each level. This condition causes Indonesian textbooks not to provide separate task-level categories, but rather leave it entirely to teachers to adjust them. As a result, the potential for learning obstacles is greater because assignments tend to be aimed at low- to middle-ability students, while the cognitive needs of high-ability students are less facilitated.

Curriculum differences also affect the order of the topics. Singapore teaches the quadratic function before the quadratic equation, while Indonesia does the opposite. The Singapore sequence is more in line with the epistemological structure of mathematics, where the concept of functions is a general idea that should be understood before studying equations as special cases. Sequence inconsistencies in Indonesia have the potential to pose didactic

obstacles, as seen in the findings that students often treat quadratic functions like quadratic equations and make incorrect simplifications.

In addition, technical analysis shows that Singapore textbooks have directed students towards the process of abstraction through the dominance of symbolic techniques, while Indonesian textbooks are still dominated by numerical and formal techniques. This indicates that Singaporean students are more geared towards generalizations and structural understanding, while Indonesian students are more likely to interact with procedures and calculations.

Overall, it can be concluded that the difference in the quality of presentation of quadratic function material between the two textbooks is mainly influenced by the curriculum design of each country. The Singapore curriculum creates conditions that are more conducive to learning differentiation and the development of mathematical abstraction, while the Indonesian curriculum requires teachers to work harder to identify learning obstacles, adjust the topic, and ensure the continuity of the epistemological flow in the learning process. These findings confirm the importance of synchronization between curriculum, assessment, and textbook design in order to support students' relational understanding and cognitive development optimally.

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